MathCon
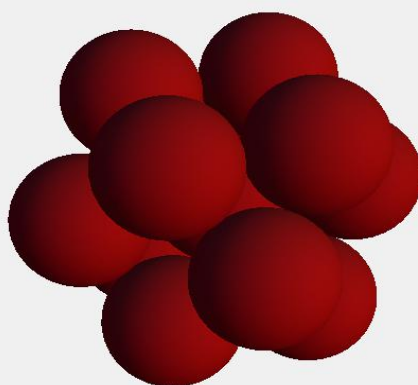*The Mathematics Firm*

Factorización

Problemas sobre factorización

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Contenido

Capítulo 1

Factorización

1.1. Factorizar las siguientes expresiones algebraicas como producto de un binomios y un trinomio.

1. $48 + 8x + 14y + xy + y^2$.   Sol: $(8 + y)(6 + x + y)$.

2. $-28 - 4x - 4xy + 7z + xz + xyz$.   Sol: $(7 + x + xy)(-4 + z)$.

3. $18 - 6x - 3xy + x^2y - 3yz + xyz$.   Sol: $(-3 + x)(-6 + xy + yz)$.

4. $10x^2 + x^3y + 10xz + 10yz + x^2yz + xy^2z$.   Sol: $(10 + xy)(x^2 + xz + yz)$.

5. $18 - 9z - 3yz + z^2 + yz^2$.   Sol: $(-3 + z)(-6 + z + yz)$.

6. $16 + 8x^2 + 2xy + x^3y + 8z + xyz$.   Sol: $(8 + xy)(2 + x^2 + z)$.

7. $-27 - 3x + 9x^2 + x^3 + 9y + xy$.   Sol: $(9 + x)(-3 + x^2 + y)$.

8. $x^3 + x^4 + xy + x^2y + x^2z + x^3z$.   Sol: $x(1 + x)(x^2 + y + xz)$.

9. $-48 + 8x - 6x^2 + x^3 - 6xy + x^2y$.   Sol: $(-6 + x)(8 + x^2 + xy)$.

10. $15 + 5x^2 + 5y + 3z + x^2z + yz$.   Sol: $(3 + x^2 + y)(5 + z)$.

11. $xy + x^2y + y^2 + xy^2 + xyz + y^2z$.   Sol: $y(x + y)(1 + x + z)$.

12. $-6xy + xy^2 + x^2y^2 - 6z + yz + xyz$.   Sol: $(-6 + y + xy)(xy + z)$.

13. $-12 - 4x^2 + 3xy + x^3y + 3xz + x^3z$.   Sol: $(3 + x^2)(-4 + x(y + z))$.

14. $12 + 4x + 4y + 3z + xz + yz$.   Sol: $(3 + x + y)(4 + z)$.

15. $xy + xy^2 + x^2z + xyz + x^2yz + xy^2z$.   Sol: $x(1 + y)(y + xz + yz)$.

16. $-48 - 6xy - 6z + 8yz + xy^2z + yz^2$.   Sol: $(8 + xy + z)(-6 + yz)$.

17. $56 + 8x + 7y + 9xy + xy^2$.   Sol: $(8 + y)(7 + x + xy)$.

18. $4 + 2y + 4z + yz + z^2$.   Sol: $(2 + z)(2 + y + z)$.

19. $56 + 7y + 8xy + xy^2 + 7z + xyz$.   Sol: $(7 + xy)(8 + y + z)$.

20. $4xy + xy^2 + 4yz + xyz + y^2z + yz^2$.   Sol: $y(x + z)(4 + y + z)$.

21. $-50 - 5x^2 + 10xy + x^3y - 5z + xyz.$ Sol: $(-5 + xy)(10 + x^2 + z).$

22. $-16 + 8x - 2y + xy + 8yz + y^2z.$ Sol: $(8 + y)(-2 + x + yz).$

23. $4x + 4y + xz + x^2z + yz + xyz.$ Sol: $(x + y)(4 + z + xz).$

24. $18 + 9x^2 + 2z + x^2z + 2yz + x^2yz.$ Sol: $(2 + x^2)(9 + z + yz).$

25. $35 + 12xy + x^2y^2 + 5z + xyz.$ Sol: $(5 + xy)(7 + xy + z).$

26. $12 - 4x - 4z - 3yz + xyz + yz^2.$ Sol: $(-3 + x + z)(-4 + yz).$

27. $-60 + 10x + 10z - 6yz + xyz + yz^2.$ Sol: $(-6 + x + z)(10 + yz).$

28. $-60 + 4xy + x^2y^2 + 10yz + xy^2z.$ Sol: $(10 + xy)(-6 + xy + yz).$

29. $81 + 9xy + 9xz + 9yz + x^2yz + xyz^2.$ Sol: $(9 + xz)(9 + xy + yz).$

30. $25 + 10y + 5xy + y^2 + xy^2.$ Sol: $(5 + y)(5 + y + xy).$

31. $3x^2 + x^3 + 3y + xy + x^2z + yz.$ Sol: $(x^2 + y)(3 + x + z).$

32. $18 + 6y + 3xz + 3yz + xyz + y^2z.$ Sol: $(3 + y)(6 + xz + yz).$

33. $-20 + 10x^2 - 2xy + x^3y + 10z + xyz.$ Sol: $(10 + xy)(-2 + x^2 + z).$

34. $-18 - 2x + 9x^2 + x^3 - 2xz + x^3z.$ Sol: $(-2 + x^2)(9 + x + xz).$

35. $-20 + 4z - 5xz - 5yz + xz^2 + yz^2.$ Sol: $(-5 + z)(4 + xz + yz).$

36. $6 + 3x + 2y + xy + 2z + xz.$ Sol: $(2 + x)(3 + y + z).$

37. $-20 - 5x^2 + 4y + x^2y - 5z + yz.$ Sol: $(-5 + y)(4 + x^2 + z).$

38. $-8 - 2y + 2xy + y^2 + xy^2.$ Sol: $(2 + y)(-4 + y + xy).$

39. $-6y + x^2y - 6xz + x^3z - 6yz + x^2yz.$ Sol: $(-6 + x^2)(y + xz + yz).$

40. $-5z - 5xz + yz + xyz + yz^2 + xyz^2.$ Sol: $(1 + x)z(-5 + y + yz).$

41. $-12 - 3xy + 4xz - 3yz + x^2yz + xyz^2.$ Sol: $(-3 + xz)(4 + xy + yz).$

42. $-x + x^2y - z - xz + xyz + x^2yz.$ Sol: $(-1 + xy)(x + z + xz).$

43. $-8 - 2x^2 + 4z + x^2z + 4yz + x^2yz$ Sol: $(4 + x^2)(-2 + z + yz).$

44. $-2y + x^2y - 2yz + x^2yz + y^2z + y^2z^2$ Sol: $y(1 + z)(-2 + x^2 + yz).$

45. $-25 - 5x + 5x^2 + x^3 - 5xz + x^3z$ Sol: $(-5 + x^2)(5 + x + xz).$

46. $42 + 6x + 6x^2 + 7xy + x^2y + x^3y$ Sol: $(7 + x + x^2)(6 + xy).$

47. $70 + 7x + 7x^2 + 10z + xz + x^2z$ Sol: $(10 + x + x^2)(7 + z).$

48. $-27 + 9y - 3xy + xy^2 - 3yz + y^2z$ Sol: $(-3 + y)(9 + xy + yz).$

49. $-14 - 2x^2 + 7y + x^2y - 2z + yz$ Sol: $(-2 + y)(7 + x^2 + z).$

50. $-3x + x^2 - 3z + xz + xyz + yz^2$ Sol: $(x + z)(-3 + x + yz).$

51. $ax^4 + ay^4 + x^6z^2 + x^2y^4z^2 + ayz^3 + x^2yz^5$ Sol: $(a + x^2z^2)(x^4 + y^4 + yz^3).$

52. $-8 + 2x^2y^2 - 4yz + x^2y^3z + 2z^4 + yz^5$ Sol: $(2 + yz)(-4 + x^2y^2 + z^4).$

53. $9c + 9x + cx^3 + x^4 + cx^2z + x^3z$ Sol: $(c + x)(9 + x^3 + x^2z).$

1.1. Factorizar las siguientes expresiones algebraicas como producto de un binomios y un trinomio. 4

54. $4 - 4x^2y - yz - xy^2z + x^2y^2z + x^3y^3z$ Sol: $(-1 + x^2y)(-4 + yz + xy^2z)$.

55. $3a + ax^2y + axy^3 + 3xz^3 + x^3yz^3 + x^2y^3z^3$ Sol: $(3 + x^2y + xy^3)(a + xz^3)$.

56. $-48 - 6a + 8x^3y + ax^3y - 6x^2y^2 + x^5y^3$ Sol: $(-6 + x^3y)(8 + a + x^2y^2)$.

57. $9x^4 + x^8 + 9y^3 + x^4y^3 + 9z^4 + x^4z^4$ Sol: $(9 + x^4)(x^4 + y^3 + z^4)$.

58. $-5z + cz - 5xz + cxz + yz^2 + xyz^2$ Sol: $(1 + x)z(-5 + c + yz)$.

59. $-3x^3z + ax^3z - 3z^3 + az^3 + x^5z^3 + x^2z^5$ Sol: $z(x^3 + z^2)(-3 + a + x^2z^2)$.

60. $16 + 4x^3z + 4x^2yz + x^5yz^2 + 4xz^3 + x^3yz^4$ Sol: $(4 + x^2yz)(4 + x^3z + xz^3)$.

61. $x^3y + x^4y + x^5z^2 + x^6z^2 + xyz^2 + x^3z^4$ Sol: $x(x^2 + x^3 + z^2)(y + x^2z^2)$.

62. $12 + 4z + 3x^2yz + x^2yz^2 + 4yz^3 + x^2y^2z^4$ Sol: $(4 + x^2yz)(3 + z + yz^3)$.

63. $bx^4 + bxy^2 + x^6yz + x^3y^3z + bxz^3 + x^3yz^4$ Sol: $x(b + x^2yz)(x^3 + y^2 + z^3)$.

64. $1 - xz - x^3z - x^2yz + x^4z^2 + x^3yz^2$ Sol: $(-1 + xz)(-1 + x^3z + x^2yz)$.

65. $-54 - 6a - 6x^2z^2 + 9xz^3 + axz^3 + x^3z^5$ Sol: $(9 + a + x^2z^2)(-6 + xz^3)$.

66. $-15 - 3x^2y^2 + 5x^3z + x^5y^2z - 3xz^2 + x^4z^3$ Sol: $(-3 + x^3z)(5 + x^2y^2 + xz^2)$.

67. $-28 + 7x^3 - 4xy^2 + x^4y^2 - 4z^3 + x^3z^3$ Sol: $(-4 + x^3)(7 + xy^2 + z^3)$.

68. $7c + cx^2y^2 + 7x^2z + x^4y^2z + cy^3z + x^2y^3z^2$ Sol: $(c + x^2z)(7 + x^2y^2 + y^3z)$.

69. $8x + x^2 + 8xyz^2 + x^2yz^2 + xyz^3 + xy^2z^5$ Sol: $x(1 + yz^2)(8 + x + yz^3)$.

70. $9b + 9x^3y + bx^2z + x^5yz + 9z^3 + x^2z^4$ Sol: $(9 + x^2z)(b + x^3y + z^3)$.

71. $36 - 6z - 6x^2z^2 + x^2z^3 - 6z^4 + z^5$ Sol: $(-6 + z)(-6 + x^2z^2 + z^4)$.

72. $-3 - y^4 + 3z + 3x^2yz + y^4z + x^2y^5z$ Sol: $(3 + y^4)(-1 + z + x^2yz)$.

73. $-4x^2y + x^3y - 4y^3 + xy^3 - 4y^3z + xy^3z$ Sol: $(-4 + x)y(x^2 + y^2(1 + z))$.

74. $4x^3 + x^3y + 4xy^3 + xy^4 + x^6z + x^4y^3z$ Sol: $x(x^2 + y^3)(4 + y + x^3z)$.

75. $-x^4 + x^5z^2 + x^6z^2 - yz^3 + xyz^5 + x^2yz^5$ Sol: $(-1 + xz^2 + x^2z^2)(x^4 + yz^3)$.

76. $10c + 10y + 10y^3z + cz^3 + yz^3 + y^3z^4$ Sol: $(c + y + y^3z)(10 + z^3)$.

77. $-7 - x^3y + 7y^3 + x^3y^4 - xy^2z + xy^5z$ Sol: $(-1 + y^3)(7 + x^3y + xy^2z)$.

78. $cx^3y + cy^4 + x^5yz^2 + x^2y^4z^2 + x^3yz^4 + y^4z^4$ Sol: $y(x^3 + y^3)(c + x^2z^2 + z^4)$.

79. $4xz^2 + 4x^2z^2 + 4z^3 + xz^6 + x^2z^6 + z^7$ Sol: $z^2(x + x^2 + z)(4 + z^4)$.

80. $6x^2z + 6xy^2z + x^3z^2 + x^2y^2z^2 + 6yz^3 + xyz^4$ Sol: $z(6 + xz)(x^2 + xy^2 + yz^2)$.

81. $2x + x^4 + 2xy^2 + x^4y^2 + xz^3 + xy^2z^3$ Sol: $x(1 + y^2)(2 + x^3 + z^3)$.

82. $x^5yz + x^4y^3z + x^4yz^3 + x^2yz^4 + xy^3z^4 + xyz^6$ Sol: $xyz(x + y^2 + z^2)(x^3 + z^3)$.

83. $-4x^3z + x^7z + x^3y^4z - 4yz^3 + x^4yz^3 + y^5z^3$ Sol: $(-4 + x^4 + y^4)z(x^3 + yz^2)$.

84. $-4 - x^2y + 4y^3 + x^2y^4 + 4x^2yz + x^4y^2z$ Sol: $(4 + x^2y)(-1 + y^3 + x^2yz)$.

85. $4x^2y^2 + 4y^3 + 4x^3z + x^2y^2z^3 + y^3z^3 + x^3z^4$ Sol: $(x^2y^2 + y^3 + x^3z)(4 + z^3)$.

86. $bxyz + bx^2z^2 + bxyz^2 + x^2y^2z^2 + x^3yz^3 + x^2y^2z^3$ Sol: $xz(y + xz + yz)(b + xyz)$.

87. $6y^3 + x^2y^5 + 6x^3z + x^5y^2z + 6yz^3 + x^2y^3z^3$ Sol: $(6 + x^2y^2)(y^3 + x^3z + yz^3)$.

88. $28 + 7b + 7xy + 4yz^3 + byz^3 + xy^2z^3$ Sol: $(4 + b + xy)(7 + yz^3)$.

89. $x^5 + x^6y + x^4y^3 + x^2yz^2 + x^3y^2z^2 + xy^4z^2$ Sol: $x(x + x^2y + y^3)(x^3 + yz^2)$.

90. $6 - 3a - 2xyz + axyz - 2y^3z + ay^3z$ Sol: $(-2 + a)(-3 + xyz + y^3z)$.

91. $x^5y^2z + x^4y^3z + x^3y^4z + x^2y^5z + x^2y^4z^2 + y^6z^2$ Sol: $y^2(x^2 + y^2)z(x^3 + x^2y + y^2z)$.

92. $-50 - 5xy + 10y^4 + xy^5 - 5x^2yz + x^2y^5z$ Sol: $(-5 + y^4)(10 + xy + x^2yz)$.

93. $-6b - 6x^3y + byz^3 + x^3y^2z^3 + bz^4 + x^3yz^4$ Sol: $(b + x^3y)(-6 + yz^3 + z^4)$.

94. $40 + 10c + 10y^3 + 4xz + cxz + xy^3z$ Sol: $(4 + c + y^3)(10 + xz)$.

95. $x^3z + x^3y^2z + xz^2 + x^2yz^2 + xy^2z^2 + x^2y^3z^2$ Sol: $x(1 + y^2)z(x^2 + z + xyz)$.

96. $10x + x^2yz + x^3yz + 10x^2z^2 + x^3yz^3 + x^4yz^3$ Sol: $x(10 + xyz + x^2yz)(1 + xz^2)$.

97. $-15 + 3x^2 - 5x^3z + x^5z - 5xz^3 + x^3z^3$ Sol: $(-5 + x^2)(3 + x^3z + xz^3)$.

98. $60 + 6x + 10y^4 + xy^4 + 10x^3z + x^4z$ Sol: $(10 + x)(6 + y^4 + x^3z)$.

99. $30 + 3y^3 + 10y^4 + y^7 + 3xyz^2 + xy^5z^2$ Sol: $(3 + y^4)(10 + y^3 + xyz^2)$.

100. $63 + 9x^2y + 9x^3y + 7xz^3 + x^3yz^3 + x^4yz^3$ Sol: $(7 + x^2y + x^3y)(9 + xz^3)$.

101. $3xy^3 + x^4y^3z + 3xz^2 + 3xyz^2 + x^4z^3 + x^4yz^3$ Sol: $x(3 + x^3z)(y^3 + z^2 + yz^2)$.

102. $-6y^3 + cy^3 - 6xy^3 + cxy^3 + y^3z + xy^3z$ Sol: $(1 + x)y^3(-6 + c + z)$.

103. $bx^2 + x^3 + bx^3z + x^4z + bxz^3 + x^2z^3$ Sol: $x(b + x)(x + x^2z + z^3)$.

104. $-3x^3z - 3y^3z + x^3y^4z + y^7z + x^5yz^2 + x^2y^4z^2$ Sol: $(x^3 + y^3)z(-3 + y^4 + x^2yz)$.

105. $x^4y^2 + x^2y^5 + x^6z + x^4y^3z + x^4z^3 + x^2y^3z^3$ Sol: $x^2(x^2 + y^3)(y^2 + x^2z + z^3)$.

106. $9y^3z + x^3y^4z + y^6z + 9z^4 + x^3yz^4 + y^3z^4$ Sol: $(9 + x^3y + y^3)z(y^3 + z^3)$.

107. $6yz + yz^2 + xy^3z^2 + 6z^4 + z^5 + xy^2z^5$ Sol: $z(6 + z + xy^2z)(y + z^3)$.

108. $8x^3z + bx^3z + 8x^2z^2 + bx^2z^2 + 8z^3 + bz^3$ Sol: $(8 + b)z(x^3 + x^2z + z^2)$.

109. $3a + 3xy + ax^3y + x^4y^2 + axyz^2 + x^2y^2z^2$ Sol: $(a + xy)(3 + x^3y + xyz^2)$.

110. $-2x^2y^2 + x^3y^2z^2 - 2z^3 - 2z^4 + xz^5 + xz^6$ Sol: $(-2 + xz^2)(x^2y^2 + z^3 + z^4)$.

111. $by + cy + bxyz + cxyz + bx^2yz + cx^2yz$ Sol: $(b + c)y(1 + xz + x^2z)$.

112. $-20 - 4c + 5x^2y + cx^2y + 5yz^3 + cyz^3$ Sol: $(5 + c)(-4 + x^2y + yz^3)$.

113. $x^4y + x^5z + x^3y^2z + x^4yz^2 + x^5z^3 + x^3y^2z^3$ Sol: $x^3(xy + x^2z + y^2z)(1 + z^2)$.

114. $-5x^4 + x^7y + x^6y^2 - 5xy^2z + x^4y^3z + x^3y^4z$ Sol: $x(-5 + x^3y + x^2y^2)(x^3 + y^2z)$.

115. $3xz + x^3yz + 3xy^2z + x^3y^3z + x^2y^2z^2 + x^2y^4z^2$ Sol: $x(1 + y^2)z(3 + x^2y + xy^2z)$.

116. $6y^3 + 6z + xy^4z^2 + 6yz^3 + xyz^3 + xy^2z^5$ Sol: $(6 + xyz^2)(y^3 + z + yz^3)$.

117. $-3x^2y^2 + x^6y^2 - 3z + x^4z - 3xyz + x^5yz$ Sol: $(-3 + x^4)(x^2y^2 + z + xyz)$.

118. $2xy + 2x^3z + 2yz + x^3yz^2 + x^5z^3 + x^2yz^3$ Sol: $(xy + x^3z + yz)(2 + x^2z^2)$.

119. $bx + by^4 + x^3yz + x^2y^5z + bxyz^2 + x^3y^2z^3$ Sol: $(b + x^2yz)(x + y^4 + xyz^2)$.

120. $yz + x^3yz + x^2y^2z + xyz^3 + x^4yz^3 + x^3y^2z^3$ Sol: $y(1 + x^3 + x^2y)z(1 + xz^2)$.